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# Influence of Linear Profile Modification and Loading Conditions on the Dynamic Tooth Load and Stress of High Contact Ratio Gears

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DYNAMIC TOOTH LOAD AND STRESS OF HIGH CONTACT RATIO SPUR GEARS

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ABSTRACT

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This paper presents a computer simulation for the dynamic response of high-contact-ratio spur gear transmissions. High contact ratio gears have the potential to produce lower dynamic tooth loads and minimum root stress but they can be sensitive to tooth profile errors. The analysis presented in this paper examines various profile modifications under realistic loading conditions. The effect of these modifications on the dynamic load (force) between mating gear teeth and the dynamic root stress is presented. Since the contact stress is dependent on the dynamic load, minimizing dynamic loads will also minimize contact stresses.

This paper shows that the combination of profile modification and the applied load (torque) carried by a gear system has a significant influence on gear dynamics. The ideal modification at one value of applied load will not be the best solution for a different load. High-contact-ratio gears were found to require less modification than standard low-contact-ratio gears. High-contact-ratio gears are more adversely affected by excess modification than by under modification. In addition, the optimal profile modification required to minimize the dynamic load (hence the contact stress) on a gear tooth differs from the optimal modification required to minimize the dynamic root (bending) stress.

Computer simulation can help find the design tradeoffs to determine the best profile modification to satisfy the conflicting constraints of minimizing both the load and root stress in gears which must operate over a range of applied loads.

NOMENCLATURE

$C_g$  damping coefficient of gear tooth mesh, N-sec (lb-sec)

$C_{s1}, C_{s2}$  damping coefficient of shaft, N-m-sec (lb-in.-sec)

$E_d$  gear error due to tooth deflection by load application, mm (in.)

$E_p$	tooth profile error or modification. $E_p$ is positive if material was removed at the contact point, mm (in.)
$E_s$	gear error due to tooth spacing variation error. $E_s$ is positive if tooth spacing for gear 1 is less than base pitch and tooth spacing for gear 2 is greater than base pitch.
$E_t$	static transmission error of a meshing gear pair, mm (in.) $E_t$ is positive if gear 1 leads gear 2.
$F$	face width of the gear tooth, mm (in.)
$h_L$	tooth thickness at the point of load application, mm (in.)
$h_S$	tooth thickness at the point of maximum root stress, mm (in.)
$J_{L, JM}$	polar moment of inertia of load, motor, kg-mm <sup>2</sup> (in.-lb-sec <sup>2</sup> )
$J_{1, J_2}$	polar moment of inertia of gear, kg-mm <sup>2</sup> (in.-lb-sec <sup>2</sup> )
$K_d$	dynamic factor
$K_g$	stiffness of gear tooth, N/mm (lb/in.)
$K_{s1}, K_{s2}$	stiffness of shaft, N-mm/rad (in.-lb/rad)
$L_n$	normalized length of tooth profile modification zone defined such that $L_n = 1.0$ is the length from tooth tip to HP2OTC, measured along the line of contact
$l_s$	distance between load point and the point of maximum root stress, mm (in.)
$Q^a$	combined meshing compliance of tooth pair a, mm/N (in./lb)

$Q_b$	combined meshing compliance of tooth pair b, mm/N (in./lb)
$Q_c$	combined meshing compliance of tooth pair c, mm/N (in./lb)
$q_{bj}$	tooth deflection due to bending, shear, and axial deflections, mm (in.)
$q_{fj}$	tooth deflection due to the flexibility of fillet and tooth foundation, mm (in.)
$q_{cj}$	local tooth deflection due to the contact stresses, mm (in.)
$q_{j1}, q_{j2}$	total deflection of a single tooth, mm (in.)
$R_{b1}, R_{b2}$	base radius, mm (in.)
$r$	tooth fillet radius, mm (in.)
$S_n$	ratio of maximum static root stress at an applied load to the maximum static root stress at the design load for unmodified gears
$T_{f1}, T_{f2}$	frictional torque on gear, N-mm (in./lb)
$T_L$	output torque on load, N-mm (in./lb)
$T_M$	input torque on motor, N-mm (in./lb)
$t$	time, s
$W$	total transmitted load, N (lb)
$W_a$	transmitted load shared by tooth pair a, N (lb)
$W_b$	transmitted load shared by tooth pair b, N (lb)
$W_c$	transmitted load shared by tooth pair c, N (lb)
$W_d$	dynamic tooth load, N (lb)
$W_n$	normalized total transmitted load
$\beta_j$	angle between the transmitted load and a line perpendicular to the tooth center line, deg
$\gamma_s$	angle defining the location of maximum tooth root stress, deg
$\Delta$	amount of profile modification (thickness of material removed from tip of involute gear tooth), defined such that $\Delta = 1.0$ is the minimum amount of tip relief recommended by Welbourn, mm
$\delta$	gear tooth backlash, mm (in.)
$\theta_L$	angular displacement of load, rad
$\theta_M$	angular displacement of motor, rad
$\theta_1, \theta_2$	angular displacement of gear, rad
$\dot{\theta}$	angular velocity, rad/sec
$\ddot{\theta}$	angular acceleration, rad/sec <sup>2</sup>

$\xi_g$	damping ratio of gear mesh
$\xi_s$	damping ratio of shafts
$\sigma$	gear tooth stress, MPa (kpsi)

Subscripts:

1	driving gear
2	driven gear

## INTRODUCTION

Recently, there has been growing interest in using high contact ratio spur gears for improved gear transmission design. Most present day spur gearing is low contact ratio, operating with contact ratios of 1.3 to 1.6. Contact ratio is defined as the average number of tooth pairs in contact under static conditions, and without errors and tooth profile modifications. High contact ratio gears (HCRG) operate with a contact ratio greater than two. This means there are at least two tooth pairs in contact at all times during the gear mesh. Because the transmitted load is always shared by at least two tooth pairs for HCRG, the individually shared tooth load tends to be less than that for low contact ratio gears (LCRG). The lower shared tooth load in HCRG decreases tooth root (bending) stress and contact stress, and potentially increases load-carrying capacity without substantially increasing the weight for power transmissions.

Although HCRG can provide a higher power-to-weight ratio than LCRG, HCRG are expected to be dynamically more sensitive to tooth errors and profile modifications due to multiple tooth contact. A major concern in gearing is the dynamic load and stress that the gear teeth experience in actual operation. High dynamic load and stress can lead to detrimental effects such as gear noise, tooth fatigue, and surface failure. This dynamic effect can be reduced by applying proper tooth profile modifications to the gear set. The amount and length of profile modification are determined according to a given design torque, usually the maximum applied torque. Tooth profile modification is regarded as one of the most effective ways to reduce dynamics and vibration of gear systems, however, when a modified gear system operates at other than the design torque, dynamic effect may become significant. The effect of tooth profile modification on LCRG dynamics has been investigated extensively (1-9). Much less work has been done for HCRG (7-9). In order to utilize HCRG designs more effectively, it is necessary to perform an in-depth study of the dynamic behavior of HCRG taking into account the tooth profile modifications and loading conditions.

This paper presents a computer-aided analysis of the influence of linear tooth profile modification and applied loading on the dynamic response of an HCRG transmission. A computer program developed previously for LCRG (5,6) was extended to perform the analysis for HCRG. The program has the capabilities to define and modify the gear tooth profile geometry, to calculate tooth deformation under load, and to determine the critical stress at the tooth root. Transient dynamic motions and natural frequencies of a HCRG transmission are solved using the program. The analysis procedure includes varying the total amount and length of profile modification systematically to determine their effects on the dynamic load and stress of a HCRG system operating at various applied loads. Contact stresses are not

calculated by the computer program discussed in this paper. However, since the contact stress in gear teeth is directly dependent on the force between mating teeth, a gear design which minimizes the dynamic load will also have minimum dynamic contact stress. The influence of tooth profile modification and of the operating load are presented and discussed.

It was found that the dynamic load and dynamic stress of HCRG are affected significantly by the length and amount of profile modification. The optimum profile modification to minimize the dynamic load is different from the optimum profile modification to minimize the dynamic root stress. Improper profile modification has a more detrimental effect on dynamic tooth load than on dynamic stress. A set of HCRG operating at a constant torque can be appropriately modified to minimize dynamic response. HCRG that must operate over a range of loads can be modified differently to minimize either the dynamic loads or the dynamic stresses according to the procedure outlined in this paper.

## THEORETICAL BACKGROUND

### HCRG Transmission Model

A simple parallel shaft HCRG transmission is depicted in Fig. 1. The system consists of a pair of high-contact-ratio gears connected to a motor and a load by flexible shafts. The theoretical model assumes the motor, the load, and the two gears act as mass inertias, and the shafts and gear teeth act as springs of a rotational system. The motion of the system is expressed by the following set of differential equations:

$$J_M \ddot{\theta}_M + C_{S1}(\dot{\theta}_M - \dot{\theta}_1) + K_{S1}(\theta_M - \theta_1) = T_M \quad (1)$$

$$\begin{aligned} J_1 \ddot{\theta}_1 + C_{S1}(\dot{\theta}_1 - \dot{\theta}_M) + K_{S1}(\theta_1 - \theta_M) + C_g(t) \\ \times [R_{b1}\dot{\theta}_1 - R_{b2}\dot{\theta}_2] + K_g(t)[R_{b1}(R_{b1}\theta_1 - R_{b2}\theta_2)] \\ = T_{f1}(t) \end{aligned} \quad (2)$$

$$\begin{aligned} J_2 \ddot{\theta}_2 + C_{S2}(\dot{\theta}_2 - \dot{\theta}_1) + K_{S2}(\theta_2 - \theta_1) + C_g(t) \\ \times [R_{b2}\dot{\theta}_2 - R_{b1}\dot{\theta}_1] + K_g(t)[R_{b2}(R_{b2}\theta_2 - R_{b1}\theta_1)] \\ = -T_{f2}(t) \end{aligned} \quad (3)$$

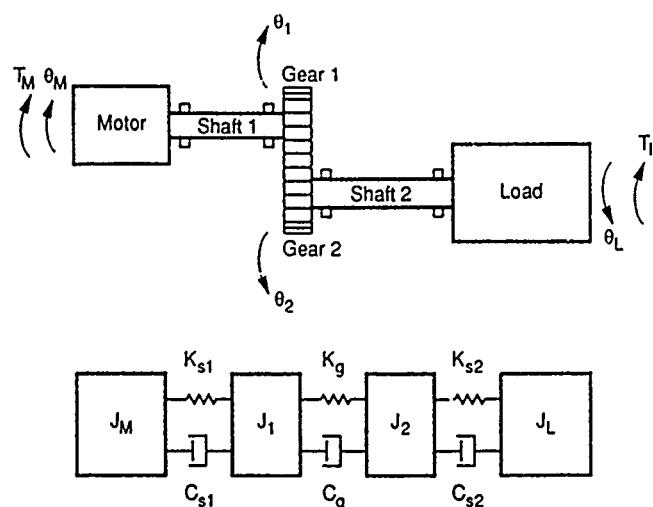


Fig. 1. Simple high-contact-ratio gear transmission system.

$$J_L \ddot{\theta}_L + C_{S2}(\dot{\theta}_L - \dot{\theta}_2) + K_{S2}(\theta_L - \theta_2) = -T_L \quad (4)$$

In developing Eqs. (1) to (4) several simplifying assumptions were employed: the dynamic process is defined in the rotating plane of the gear pair; the contact between gear teeth is assumed to be along the theoretical line of action; damping due to lubrication, etc. is expressed as a constant damping factor (ratio of the damping coefficient to the critical damping coefficient).

The stiffnesses, damping and friction, and mass moments of inertia of the system components can be found from fundamental mechanics principles. The equations of motion contain the excitation terms due to variation of gear meshing stiffness and damping. The meshing stiffness and damping are functions of the mesh point along the line of action. Detailed analyses of system component properties and dynamic motion of LCRG transmissions were presented in previous studies (10,11). Analogous procedures can be applied to HCRG. Those that are different from LCRG or of more significant nature are presented in this paper.

### Gear Meshing Stiffness

The HCRG tooth form with tangent undercut, as presented by Cornell (12), is used in the investigation. The individual tooth spring stiffness is determined by considering the tooth to be a nonuniform cantilever beam supported by the flexible fillet region and foundation. If we let  $j$  be a contact point on the tooth profile and  $W_j$  be the transmitted load, the deformation at  $j$  in the direction of  $W_j$  for a single tooth can be written as (12),

$$q_j = q_{bj} + q_{fj} + q_{cj} \quad (5)$$

and the deformation for a pair of teeth in contact is

$$q_{j12} = q_{j1} + q_{j2} \quad (6)$$

where the subscript 1 represents the driving gear and the subscript 2 represents the driven gear. The combined meshing compliance,  $Q_j$ , of a pair of meshing teeth at point  $j$  may be expressed as:

$$Q_j = q_{j12}/W_j \quad (7)$$

Variation of meshing compliance with the tooth meshing position determines various static transmission properties as well as gear meshing stiffness of the HCRG system. Figure 2 illustrates the motion of a pair of meshing gear teeth. This analysis is limited to HCRG with contact ratio between two and three. This means there will always be either two or three tooth pairs in contact. We designate four consecutive tooth pairs a to d, and begin our analysis at the moment in which a and b are in contact, and a third tooth pair c is just entering contact. The initial contact of tooth pair c occurs at point A, where the addendum circle of the driven gear intersects the line of action. As the gears rotate, the point of contact will move along the line of action APF where P is the pitch point. As tooth pair c reaches point B, the leading tooth pair a disengages at point F leaving only pairs b and c in contact. When tooth pair c reaches point C, the next tooth pair d begins engagement at A. Thus, the meshing action alternates between triple and double contact zones as shown in the figure.

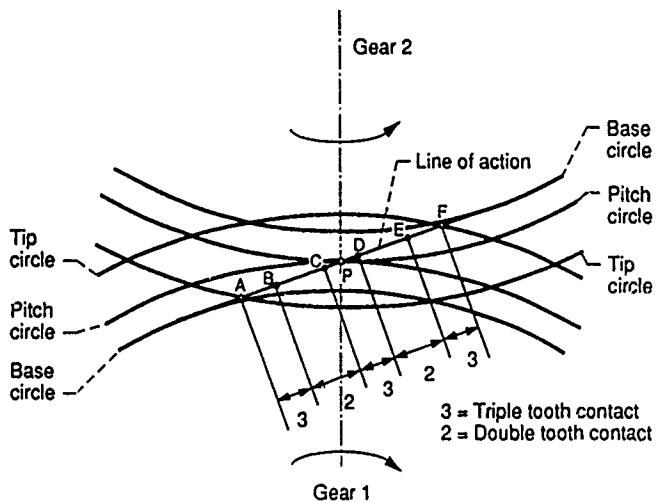


Fig. 2. Illustration of high-contact-ratio gear meshing action.

If there are three tooth pairs in contact, then the static transmission error  $E_t$ , and the shared tooth load  $W_j$ , for each individual tooth pair at contact point  $j$  may be expressed as:

$$(E_t^a)_j = (E_{d1}^a)_j + (E_{d2}^a)_j + (E_{p1}^a)_j + (E_{p2}^a)_j \quad (8)$$

$$(E_t^b)_j = (E_{d1}^b)_j + (E_{d2}^b)_j + (E_{p1}^b)_j + (E_{p2}^b)_j + (E_{s1}^b)_j + (E_{s2}^b)_j \quad (9)$$

$$(E_t^c)_j = (E_{d1}^c)_j + (E_{d2}^c)_j + (E_{p1}^c)_j + (E_{p2}^c)_j + (E_{s1}^c)_j + (E_{s2}^c)_j \quad (10)$$

$$W = W_j^a + W_j^b + W_j^c \quad (11)$$

Note: The subscript  $j$  has been used to indicate the contact point at a particular time. The position of this contact point will differ between the three tooth pairs in contact.

All the error terms above can be converted to the linear relative displacement between mating gears along the line of action. The static transmission error  $E_t$  is the total relative displacement of the driven gear with respect to the driving gear along this line. During meshing, the static transmission error of the three mating tooth pairs will be the same. Therefore, from Eqs. (8) to (10),

$$Q_j^a W_j^a + (E_p^a)_j = Q_j^b W_j^b + (E_p^b)_j + (E_s^b)_j = Q_j^c W_j^c + (E_p^c)_j + (E_s^c)_j \quad (12)$$

where

$$(E_s)_j = (E_{s1})_j + (E_{s2})_j \quad (13)$$

$$(E_p)_j = (E_{p1})_j + (E_{p2})_j \quad (14)$$

$$(E_d)_j = (E_{d1})_j + (E_{d2})_j = Q_j W_j \quad (15)$$

Solving Eqs. (11) and (12) simultaneously yields

$$W_j^a = \frac{[(E_s^c)_j + (E_s^c)_j - (E_p^a)_j]Q_j^a + [(E_p^b)_j + (E_s^b)_j - (E_s^c)_j]Q_j^b + Q_j^c W_j}{Q_j^a Q_j^b + Q_j^b Q_j^c + Q_j^a Q_j^c} \quad (16)$$

$$W_j^b = \frac{[(E_s^c)_j + (E_s^c)_j - (E_p^b)_j]Q_j^a + [(E_p^a)_j - (E_s^b)_j - (E_s^c)_j]Q_j^b + Q_j^c W_j}{Q_j^a Q_j^b + Q_j^b Q_j^c + Q_j^a Q_j^c} \quad (17)$$

$$W_j^c = \frac{[(E_p^b)_j + (E_s^c)_j - (E_p^b)_j]Q_j^a + [(E_s^b)_j - (E_s^c)_j - (E_s^c)_j]Q_j^b + Q_j^c W_j}{Q_j^a Q_j^b + Q_j^b Q_j^c + Q_j^a Q_j^c} \quad (18)$$

The gear meshing stiffness,  $K_g$ , at point  $j$  is then,

$$(K_g)_j = W_j^a / (E_t^a)_j + W_j^b / (E_t^b)_j + W_j^c / (E_t^c)_j = W / (E_t)_j \quad (19)$$

In the analyses above and those to follow, the position of the contact point  $j$  of the gear teeth along the line of action is expressed in terms of roll angle of the driving gear tooth. The transmission error and meshing stiffness for HCRG in the double contact zone can be calculated by applying similar procedures. They are the same as those developed for LCRG and can be found in Refs. 5 and 11.

#### Tooth Profile Modification

Tooth profile modification can be converted to the equivalent linear relative displacement of the mating teeth and incorporated into the  $E_p$  term in Eqs. (12) to (18). Varying the tooth profile will change gear transmission error and affect the shared tooth load and gear meshing stiffness.

A typical gear tooth showing the profiles both before and after modification is illustrated in Fig. 3(a). A sample modification chart is shown in Fig. 3(b). The straight lines on the chart present three examples of linear profile modification.

In this study, the same amount and the same length of profile modifications are applied to the tooth tip of both pinion and gear. The conventional amount of tip relief has been chosen as a reference value to normalize the amount of profile modification. This conventional amount (if no spacing error is considered) is equal to the combined tooth deflection evaluated at the highest point of second double tooth contact (HP2DTC), see Fig. 3(a). For the conventional amount of tip relief,  $\Delta = 1.00$ . The length of profile modification is designated  $L_n$ . The distance along the tooth profile from tooth tip to the HP2DTC is defined to be of unit length. The values of  $\Delta$  and  $L_n$  can be varied arbitrarily to obtain any desired combination. Figure 3(b) shows three examples of linear profile modification: (1)  $\Delta = 1.00$ ,  $L = 1.00$ ; (2)  $\Delta = 0.50$ ,  $L = 1.00$ , and (3)  $\Delta = 1.00$ ,  $L = 2.50$ . The third example represents the modification of tooth profile from tooth tip to the lowest point of second double tooth contact (LP2DTC).

#### Damping and Friction

The effect of damping in the shafts is due to the material and damping in the gear mesh is due to lubrication. The shaft damping coefficients are taken as:

$$C_{s1} = 2\xi_{s1} \sqrt{K_{s1}/(1/J_0 + 1/J_1)} \quad (20)$$

$$C_{s2} = 2\xi_{s2} \sqrt{K_{s2}/(1/J_L + 1/J_2)} \quad (21)$$

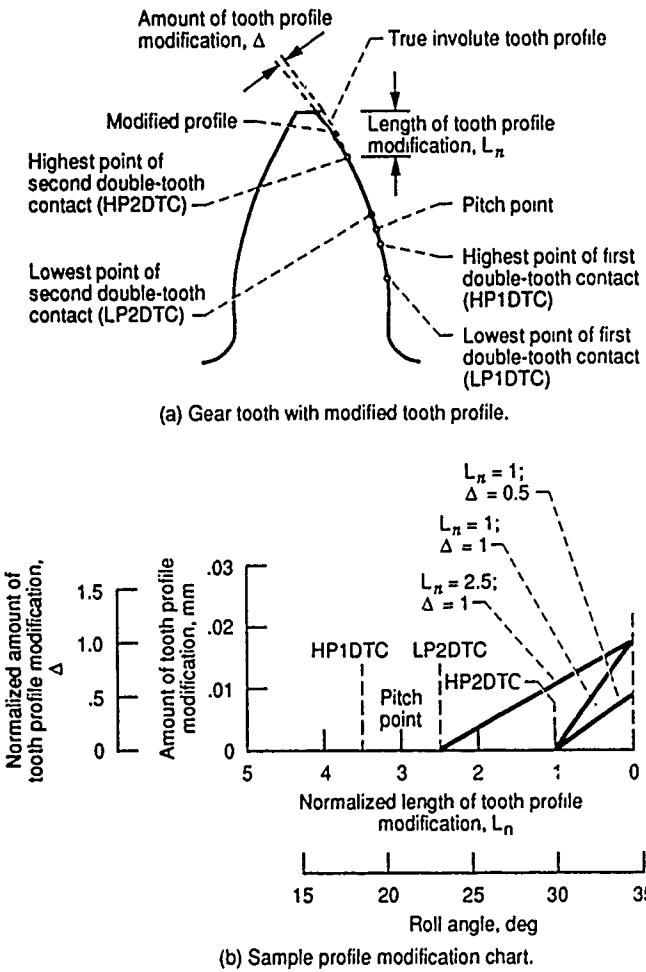


Fig. 3. Example of modified high-contact-ratio gear tooth.

and the damping in the gear mesh at contact point  $j$  is:

$$(C_g)_j = 2\xi_s \sqrt{(K_g)_j [R_{b1}^2/J_1 + R_{b2}^2/J_2]} \quad (22)$$

where  $\xi_s$  represents the damping ratio of shafts, and  $\xi_g$  the damping ratio for the gear mesh. From gear literature, typical damping ratios of 0.005 and 0.10 respectively were chosen for  $\xi_s$  and  $\xi_g$ . Friction torques,  $T_M$ ,  $T_{f1}$ ,  $T_{f2}$ , and  $T_L$  in the dynamic Eqs. (1) to (4) were determined using the procedures derived in Ref. 10.

#### Solution of Gear Dynamic Motions

The differential equations of motion are solved by a linearized iterative procedure (11). The linearized equations are obtained by dividing the mesh period into many equal intervals. In the analysis, a constant input torque  $T_M$  is assumed and the output torque can fluctuate as a result of time-varying stiffness, friction, and damping in the mesh. To start the solution process, initial values of the angular displacements are obtained by preloading the input shaft with the nominal torque carried by the system. Initial values of the angular speed are taken from the nominal system operating speed. For steady state operation and with the same tooth profile modification on both gear teeth, the angular displacement and angular speed of mating gears must be identical at the beginning and at the end of the meshing period. Therefore, the iteration procedure is as follows: the calculated values of the

angular displacement and speed after one mesh period are compared with the assumed initial values. Unless the differences between them are smaller than a preset tolerance, the procedure is repeated using the average of the initial and calculated values as new initial conditions.

In conducting the dynamic analysis, it is useful to identify the system natural frequencies (or critical speeds). The natural frequencies are obtained by solving the undamped system equations of motion. The varying gear meshing stiffnesses are replaced by an average value. The average meshing stiffness is taken as the sum of the discrete tooth meshing stiffness values of a mesh cycle divided by the number of mesh positions in the cycle (11).

#### Calculation of Dynamic Load and Stress

Dynamic tooth load is the product of the relative motions of gear teeth,  $(R_{b1}\theta_1 - R_{b2}\theta_2)$  and  $(R_{b1}\dot{\theta}_1 - R_{b2}\dot{\theta}_2)$ , at contact point  $j$  with the corresponding meshing stiffness and damping values. If gear 1 is the driving gear and  $\delta$  is the backlash, the following conditions can occur:

$$\text{Case (i)} \quad (R_{b1}\theta_1 - R_{b2}\theta_2)_j > 0$$

This is the normal operating case. The dynamic tooth load  $(W_d)_j$  at point  $j$  is then:

$$(W_d)_j = (K_g)_j(R_{b1}\theta_1 - R_{b2}\theta_2)_j + (C_g)_j(R_{b1}\dot{\theta}_1 - R_{b2}\dot{\theta}_2)_j \quad (23)$$

$$\text{Case (ii)} \quad (R_{b1}\theta_1 - R_{b2}\theta_2)_j \leq 0$$

$$\text{and} \quad |(R_{b1}\theta_1 - R_{b2}\theta_2)_j| \leq \delta$$

In this case, the gear will separate and the contact between the gears will be lost. Hence,

$$(W_d)_j = 0 \quad (24)$$

$$\text{Case (iii)} \quad (R_{b1}\theta_1 - R_{b2}\theta_2)_j < 0$$

$$\text{and} \quad |(R_{b1}\theta_1 - R_{b2}\theta_2)_j| > \delta$$

In this case, gear 2 will collide with gear 1 on the backside, then,

$$(W_d)_j = (K_g)_j(R_{b2}\theta_2 - R_{b1}\theta_1)_j + (C_g)_j(R_{b2}\dot{\theta}_2 - R_{b1}\dot{\theta}_1)_j \quad (25)$$

To calculate the dynamic tooth root stress, an improved and simplified method called the modified Heywood method is used. This method is considered to be accurate for the HCRG tooth form and gives results that agree well with both finite element analysis and test data (12). The modified Heywood formula for tooth root stress is

$$\sigma_j = \frac{(W_d)_j \cos \beta_j}{F} \left[ 1 + 0.26 \left( \frac{h_s}{2r} \right)^{0.7} \right] \left[ 6 \frac{i_s - \left( \frac{h_L \tan \beta_j}{2} \right)}{h_s^2} \right. \\ \left. + \left( \frac{0.72}{h_s} \right)^{0.5} \left( 1 - \frac{h_L}{h_s} v \tan \beta_j \right) \frac{\tan \beta_j}{h_s} \right] \quad (26)$$

where  $\nu \approx 1/4$  according to Heywood. The values of  $h_s$  and  $l_s$  are related to the gear tooth geometry, the load position, and the point of maximum stress in the fillet (see Fig. 4). The magnitude of  $\gamma_s$ , which defines the position of maximum fillet stress, varies with the fillet radius  $r$ , the load position, and the thickness of the tooth's thinnest section (12). For a typical LCRG tooth, the angle of  $30^\circ$  is considered to be a reasonable average value (12). However, for HCRG it is more appropriate to use  $20^\circ$  for an average  $\gamma_s$  angle. Reference 12 provides detailed analysis to find the  $l_s$  and  $h_s$  values.

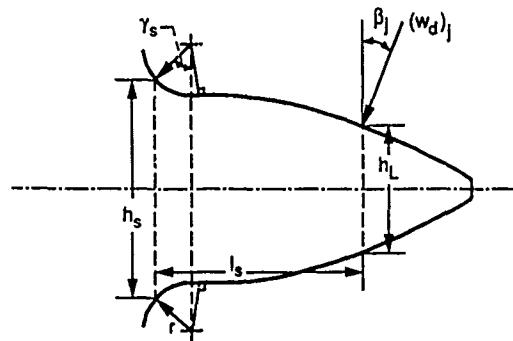


Fig. 4. Gear tooth geometry for root stress calculation.

## APPLICATION OF ANALYSIS

To apply the foregoing analysis, consider an HCRG transmission with a typical set of gears as specified in Table 1. These are identical high-contact-ratio involute spur gears with solid gear bodies. The number of teeth is 31 and the module is 3.18 (8 diametral pitch). Face width is 25.4 mm with a design load of 350 000 N/m (2000 lb/in.). The gear mesh theoretical contact ratio is 2.40. The pressure angle is  $20^\circ$ . The connecting shafts have 305 mm (12 in.) length and 25.4 mm (1 in.) diameter. Mass moments of inertia of the motor and the load are assumed to be 70 times, and 50 times the gear inertia, respectively. The material for the gears and shafts is steel.

TABLE 1. - GEAR DATA

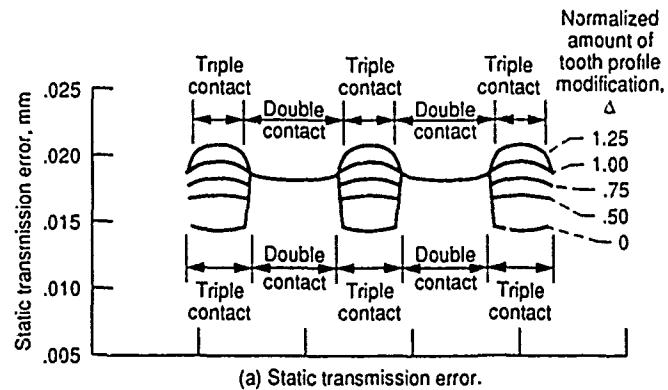
Gear tooth . . . . .	Standard involute tooth
Number of teeth . . . . .	32
Module M, mm (diametral pitch P, 1/in.) . . . . .	3.18 (8)
Pressure angle, deg . . . . .	$20^\circ$
Addendum, mm (in.) . . . . .	$0.06024 \cdot M$ (1.53/P)
Face width, mm (in.) . . . . .	25.4 (1.0)
Design torque, N/m (lb/in.) . . . . .	425 (3760)
Static tooth load, N/m (lb/in.) . . . . .	350 000 (2000)
Theoretical contact ratio . . . . .	2.40

Neglecting the rigid body mode at zero frequency, the transmission's first three natural frequencies (critical speeds) are found to be 86, 610, and 9300 rpm. Peak dynamic response of the gear transmission usually occurs at speeds near the system natural frequencies. In the following sections, the total amount of modification and the length of profile modification zone have been varied systematically to examine their effects on the peak dynamic loads and stresses of the HCRG transmission. The loading condition was also varied over a realistic range to determine its influence on the dynamics of the transmission.

### Effect of Modification Amount and Load

In this section, the length of profile modification zone is held constant at  $l_n = 1.00$  to study the

effect of the profile modification amount  $\Delta$ . Figure 5 shows that the static transmission error and shared tooth load vary significantly with the amount of modification. In this case, the applied load is the full design torque. The gear contact ratio is not affected by tip modification when the modification amount  $\Delta$  does not exceed the conventional amount of tip relief (i.e.,  $\Delta \leq 1.00$ ), however, when excess modification (such as  $\Delta = 1.25$ ) is applied, the zone of triple-tooth contact shortens and contact ratio decreases. In this case, the contact ratio is reduced from 2.40 to approximately 2.30.



(a) Static transmission error.

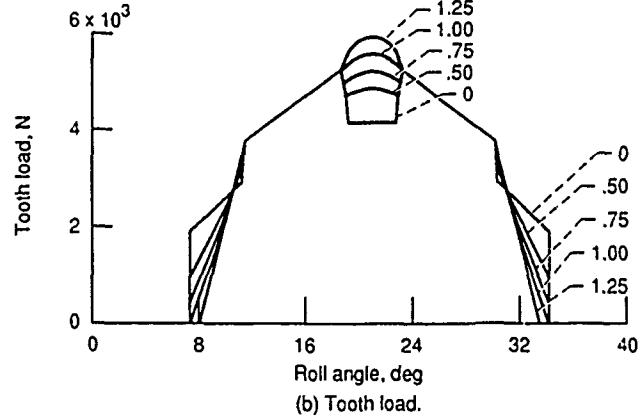


Fig. 5. Variation of static transmission error and tooth load of high-contact-ratio gear during mesh cycle.

Figure 6 shows the dynamic tooth load and dynamic tooth stress of HCRG tooth pairs as a function of the gear roll angle at the speed of 8500 rpm. This speed is approximately 90 percent of the third critical speed. Earlier analytical and experimental works have revealed that primary peak dynamic response of a gear system occurs at about 90 percent of the third critical speed (4,12). In Fig. 6, the various dashed curves show the dynamic response of gears with the modification amount  $\Delta$  at the values of 0.50, 0.75, 1.00, and 1.25. The length of modification zone is held constant at  $l_n = 1.00$ . Also, for comparison, the response of an unmodified gear pair is shown as a solid line.

Figure 6(a) shows that a small amount of modification can reduce the dynamic tooth load considerably. The lowest dynamic load in Fig. 6(a) is observed in the  $\Delta = 0.75$  case. This indicates that these high-contact-ratio gears require less than the conventional amount of profile modification. This example shows that high-contact-ratio gears require less modification than low-contact-ratio gears (see ref. 5). On the other hand, excess modification, as shown in the  $\Delta = 1.25$  case, can produce a higher dynamic load than even unmodified gears.

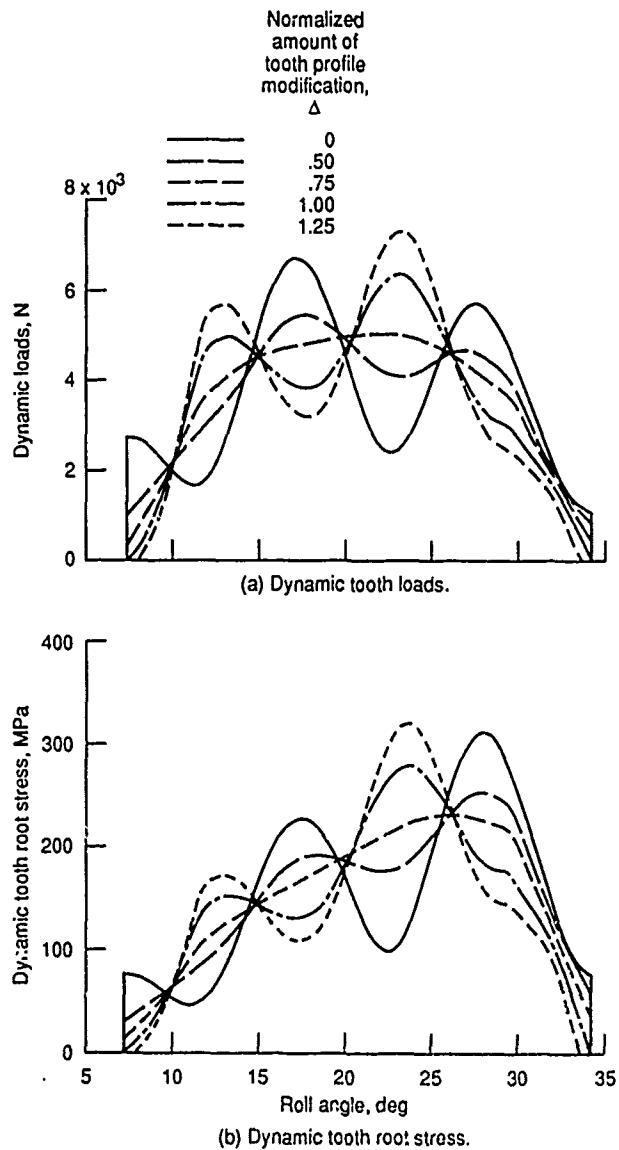


Fig. 6. Variation of high-contact-ratio gear dynamic tooth loads and dynamic tooth root stress with gear roll angle at 8500 rpm, full design load,  $L_n = 1.00$ , and varying  $\Delta$ .

Changes in tooth profile not only affect the maximum tooth load, but also the frequency of the forced dynamic response and the position on the tooth of the peak response. Both of these effects contribute to the dynamic tooth root stress curves plotted in Fig. 6(b). The proper profile modification acts to smooth the meshing action which reduces the magnitude of the gear dynamic load. It also shifts the peak load lower on the tooth. This decreases the moment of the load which minimizes the bending stress in the tooth root.

Since the peak root stress depends on both the magnitude and location of the peak tooth load, the peak load and peak stress may occur at different times during the mesh cycle. A comparative study was conducted to determine the load and stress response at varying amounts of modification over a range of speeds at a constant applied load. The dynamic load and stress responses are evaluated at 100 rpm intervals over the speed range from 2000 to 11 000 rpm. Results are presented in the form of a speed survey of dynamic load factor in Fig. 7(a) and dynamic stress factor in Fig. 7(b). The dynamic load factor is defined as the

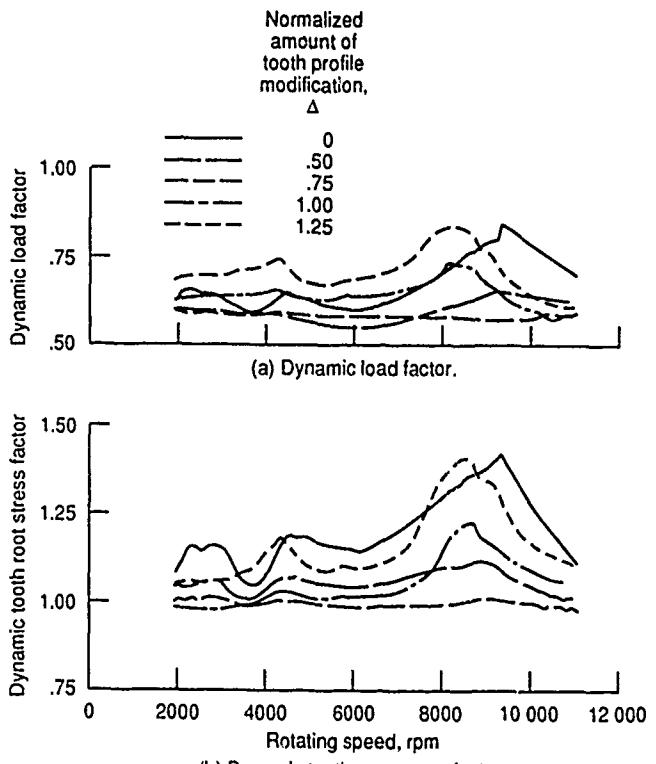


Fig. 7. Variation of high-contact-ratio gear dynamic load factor and dynamic tooth root stress factor with rotating speed at  $L_n = 1.00$ , full design load, and varying  $\Delta$ .

peak dynamic load divided by the total static load. The dynamic load factor for HCRG is typically less than unity due to load sharing by the two or more tooth pairs in mesh (8). (By comparison, the dynamic load factor for LCRG is usually greater than unity (6).) The dynamic stress factor is defined as the peak dynamic root stress divided by the peak static root stress of the unmodified case. This factor is greater than unity because the maximum dynamic stress is greater than the static tooth stress.

The solid curves in Figs. 7(a) and (b) represent the response of unmodified gears. Note that there is a prominent peak at about 9300 rpm, the primary critical speed of this HCRG transmission. Properly chosen profile modification can reduce this dynamic response considerably. The curve for  $\Delta = 0.75$  shows the lowest dynamic load factor in Fig. 7(a) and the lowest dynamic stress factor in Fig. 7(b). Over most of the speed range surveyed, the excess modification case ( $\Delta = 1.25$ ) produces more severe loads and nearly as severe stress as in unmodified gears.

Gear transmissions are generally required to operate over a range of loads due to varying power demands. Since the optimum tooth profile for one design load (torque) may not be a good solution for a different load, it is useful to investigate the dynamic performance of an HCRG transmission under various operating loads. Figure 8 summarizes data from more than 50 speed sweeps to illustrate the effect of the amount of profile modification (at constant length of modification,  $L_n = 1.00$ ) for several values of applied loads ranging from 70 to 120 percent of the design load.

Figure 8 contains design curves for choosing values of the modification amount required for minimum dynamic load and minimum dynamic stress. In Fig. 8, the normalized maximum dynamic load is defined as the product of the maximum dynamic load factor (MDLF), obtained from a speed sweep, and the normalized applied

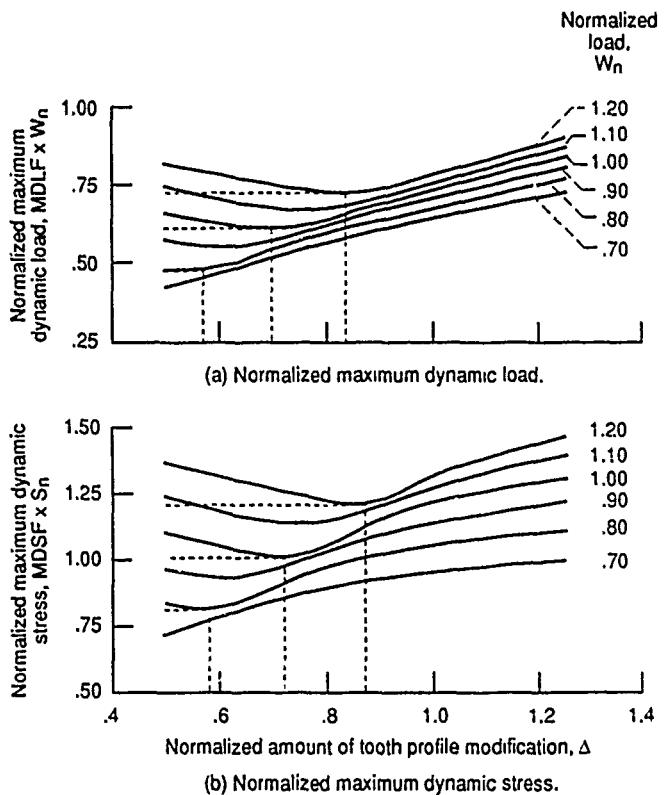


Fig. 8. Effect of amount of profile modification  $\Delta$  on normalized maximum dynamic load and normalized maximum dynamic stress at various normalized loads.

load  $W_n$ .  $W_n$  is the ratio of the applied load to the design load. The design load is defined in Table 1 as 350 000 N/m (2000 lb/in.). If the applied load equals the design load,  $W_n = 1.00$ . Likewise, the normalized maximum dynamic stress is the product of the maximum dynamic stress factor (MDSF) and the normalized static root stress  $S_n$ .  $S_n$  is the ratio of maximum static root stress at one value of an applied load to the maximum root stress at the design load for unmodified gears. These normalized values of maximum dynamic load and maximum dynamic stress are used to illustrate the absolute dynamic response of the HCRG system. The normalized parameters are useful for comparing the benefit of various tooth profile modifications at different applied loads. The actual value of the dynamic tooth load may be found by multiplying the normalized value by the value obtained at the design torque. Likewise, the actual value of the dynamic root stress may be found by multiplying the normalized value by the maximum root stress under static conditions (zero rpm) at the design torque.

Each curve in Fig. 8 is obtained by a cubic spline curve fit using seven to nine data points (each of which represents one speed sweep). The modification amount  $\Delta$  required to produce the minimum dynamic load at any single value of applied load can be read from the appropriate load curve in Fig. 8(a). Figure 8 is restricted to values of modification amount  $\Delta$  in the range 0.50 to 1.25. Since the  $W_n = 0.70$  curve has apparently not reached a minimum value at the left side of the figure, its  $\Delta$  value for minimum response will be taken to be 0.50. For the other load values considered,  $W_n = 0.80, 0.90, 1.00, 1.10$ , and 1.20, the optimum modification amounts are found to be 0.56, 0.62, 0.69, 0.75, and 0.84 respectively.

The best value of modification amount  $\Delta$  based on minimum dynamic load for any range of applied load may

be determined from Fig. 8. In Ref. 5, a method was presented for finding the best value of the modification amount  $\Delta$  to achieve minimum dynamic load for low-contact-ratio gears which must operate over a range of loads. This best value was found at the intersection of the curves corresponding to the maximum and minimum applied loads. In Fig. 8, however, the design curves for HCRG do not intersect. The procedure for finding the optimum value for a range of loads is more involved. To find the optimum value for a range of loads, the designer should plot several curves (such as in Fig. 8(a)) and find the best modification amount  $\Delta$  and the normalized maximum dynamic load for each curve. The normalized load divided by the sum of normalized loads for all curves forms a weighting function for the modification amount.

As an example, consider the load range  $W_n = 0.80$  to 1.20 in Fig. 8(a). To simplify the analysis, we consider the three load curves  $W_n = 0.80, 1.00$ , and 1.20. Values of  $\Delta$  and the corresponding normalized load for each load are found from the load curves (see the corresponding points in Fig. 8(a)). These data and calculations are shown in Table 2. The weight for each curve is calculated by the load divided by the sum of the loads. Thus for the  $W_n = 0.80$  curve, the weight is  $0.47/(0.47 + 0.59 + 0.72) = 0.264$ . This value is then multiplied by the  $\Delta$  value for this curve to produce a weighted  $\Delta$ . For  $W_n = 0.80$ , the weighted  $\Delta$  is  $0.56 \times 0.264 = 0.148$ . Finally, all of the weighted  $\Delta$  values are summed to produce the desired optimum  $\Delta$  for the load range. For our example, this optimum value's  $\Delta = 0.72$ . This is the best value of  $\Delta$  for the load range  $W_n = 0.80$  to 1.20.

TABLE 2. - EXAMPLE DATA FOR CALCULATING OPTIMUM MODIFICATION AMOUNT

$W_n$	$\Delta$	Normal maximum dynamic load	Weight	Weighted, $\Delta$
0.80	0.56	0.47	0.264	0.148
1.00	0.62	0.59	0.331	0.229
1.20	0.75	0.72	0.405	0.340
		1.78	1.000	≈ 0.72

The example above assumes an even distribution of time at each load level. If this assumption is not valid, the designer must find a time weighting factor for each  $\Delta$  value considering the relative time to be spent at each load.

Figure 8(b) can be used for choosing values of the modification amount to minimize dynamic root stress. The minimum values of the load curves  $W_n = 0.80, 0.90, 1.00, 1.10$ , and 1.20, are found to be at  $\Delta = 0.58, 0.62, 0.72, 0.75$ , and 0.87, respectively. For minimum dynamic stress in the load range  $W_n = 0.80$  to 1.20, the optimum value of  $\Delta$  is found, using the procedure described above, to be 0.74. The optimum values for  $\Delta$  based on root stress are about 3 percent higher than the optimum values based on the load. The trend of the dynamic load and the dynamic stress curves are quite similar, however, the dynamic stress curves are more sensitive to load change.

#### Effect of Modification Length and Load

The preceding discussion considered optimizing the profile modification amount  $\Delta$  with the length of modification zone fixed at the conventional value of  $L_n = 1.00$ . A similar study was performed to find the

optimum length  $L_n$  with  $\Delta$  fixed at 1.00. Figure 9 presents the dynamic tooth load and dynamic root stress of an HCRG tooth pair as a function of gear roll angle at the constant speed of 8500 rpm and at several values of  $L_n$ . The dashed curves in Fig. 9 give the dynamic response of the gears with  $L_n$  values equal to 0.50, 0.75, 1.00, 1.25, and 2.50. For comparison, the response of unmodified gears are shown as solid lines.

The lowest dynamic load is observed for the gears with  $L_n = 0.75$ ; see Fig. 9(a). The peak dynamic load for this case is very close to the static load (shown as solid line). The gears with  $L_n = 0.75$  also show the lowest value of peak dynamic stress in Fig. 9(b). The highest dynamic load and dynamic stress is observed for gears with  $L_n = 1.25$ . For the gears with  $L_n = 2.50$ , the modification zone extends from the tooth tip to the lowest point of double tooth contact (LP2DTC) as shown in Fig. 3(a). A gear tooth with this modification length will have its meshing impact at the

beginning of engagement delayed. This delay allows only a single dynamic peak occurring near the pitch point; see Fig. 9(a). The maximum dynamic load for gears with  $L_n = 1.25$  and  $L_n = 2.50$  are nearly equal, however, their maximum dynamic stress values, as shown in Fig. 9(b), differ considerably due to the difference in the position of the peak load.

To study the effect of modification length  $L_n$  on HCRG over the speed range of 2000 to 11 000 rpm, a speed survey of dynamic load factor and of dynamic stress factor is presented in Fig. 10. The response of unmodified gears is also shown for comparison. For the case studied (full design load and modification amount  $\Delta = 1.00$ ), the dynamic load and dynamic stress is lowest for gears with  $L_n = 0.75$ . The worst cases for both dynamic load and dynamic stress response are observed for unmodified gears and gears modified at  $L_n = 1.25$ . For the case of  $L_n = 2.50$ , the dynamic load is relatively high over the entire speed range, however, the dynamic stress is moderate at all speeds studied. These conclusions agree with the constant speed (8500 rpm) results of Fig. 9.

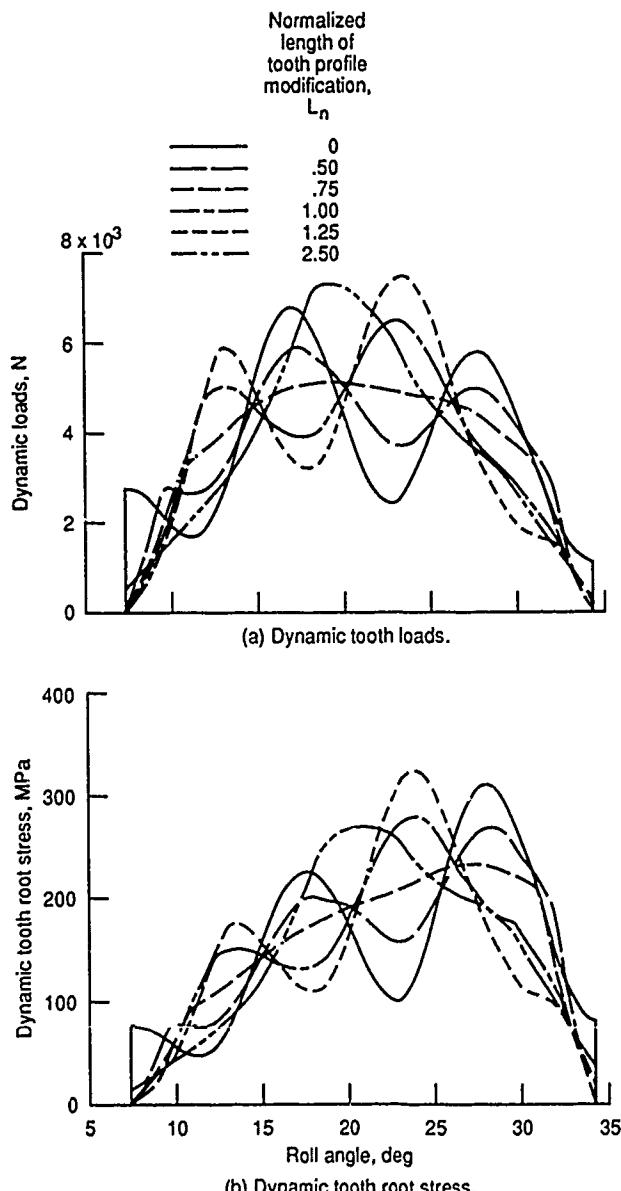


Fig. 9. Variation of high-contact-ratio gear dynamic tooth loads and dynamic tooth root stress with gear roll angle at 8500 rpm, full design load,  $\Delta = 1.00$ ; and varying  $L_n$ .

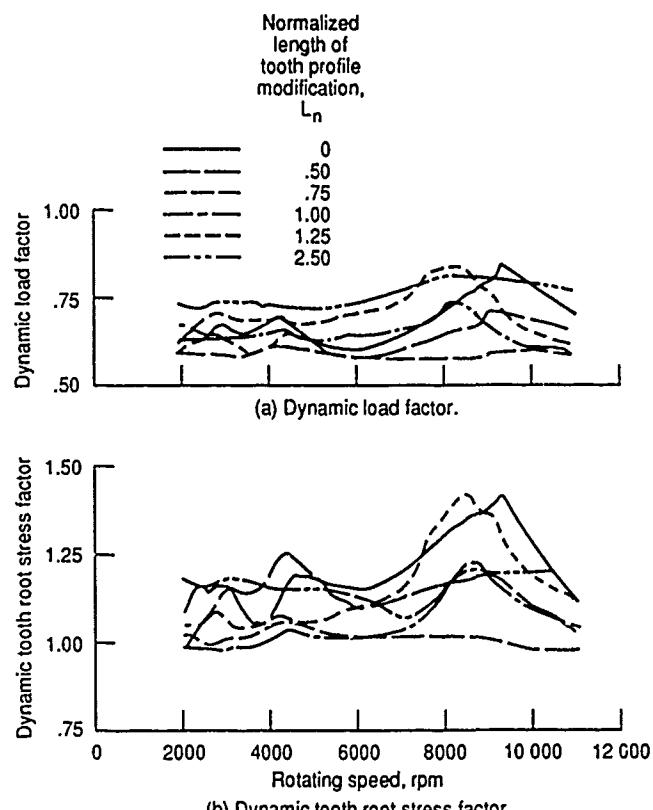
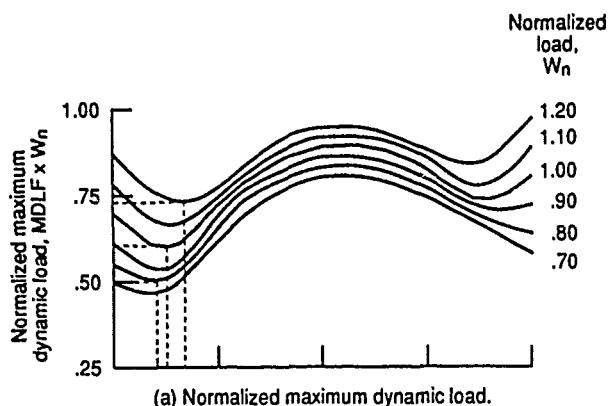
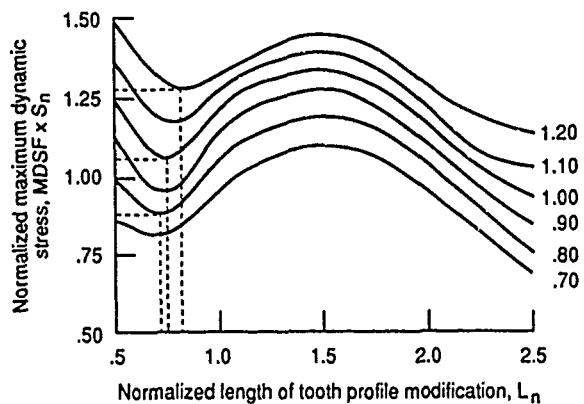


Fig. 10. Variation of high-contact-ratio gear dynamic load factor and dynamic tooth root stress factor with rotating speed at  $\Delta = 1.00$ , full design load, and varying  $L_n$ .

Figure 11 contains design curves for choosing values of the modification length  $L_n$  required for minimum dynamic tooth load and minimum dynamic root stress. These curves are similar to those in Fig. 8 and can be used in the same way. For the load values considered,  $W_n = 0.70, 0.80, 0.90, 1.00, 1.10$ , and  $1.20$ , the optimum modification lengths  $L_n$  to produce minimum dynamic load, Fig. 11(a), are found to be  $0.66, 0.69, 0.71, 0.74, 0.78$ , and  $0.82$ , respectively. For the example range of loads  $W_n = 0.80$  to  $1.20$ , the optimum  $L_n$  to minimize dynamic load is equal to  $0.76$ .



(a) Normalized maximum dynamic load.



(b) Normalized maximum dynamic stress.

Fig. 11. Effect of length of profile modification,  $L_n$  on normalized maximum dynamic load and normalized maximum dynamic stress at various normalized loads.

Likewise, Fig. 11(b) can be used for choosing values of  $L_n$  required to minimize dynamic root stress. The minimum values of the response curves of  $W_n = 0.70, 0.80, 0.90, 1.00, 1.10$ , and  $1.20$  are found to be at  $L_n = 0.70, 0.72, 0.75, 0.76, 0.80$ , and  $0.85$ , respectively. The trend of the two dynamic response curves are similar when the value of  $L_n$  is less than  $2.00$ . For the example load range of  $W_n = 0.80$  to  $1.20$ , the optimum  $L_n$  to minimize dynamic stress is found to be  $0.79$ . The optimum values of  $L_n$  for minimum dynamic stress are about 4 percent higher than that for minimum dynamic load. In this example, the excess values of  $L_n$  which reduce dynamic (root) stress but also increase dynamic load are not considered for optimum tooth profile modification.

## CONCLUSIONS

A computer simulation was conducted to investigate the effects of linear tooth profile modification on the dynamic load and tooth root stress of high-contact-ratio gears. The effects of the magnitude of modification and the length of modification zone were studied at various loads and speeds to find the optimum values to minimize dynamic load and stress. Based on results of the study, the following conclusions were obtained:

1. For any constant value of applied load (torque) carried by the gear system, computer simulation can find an optimum profile modification to minimize the dynamic tooth load and root stress for high-contact-ratio gears. This modification will not be optimum for a different value of applied load. Computer simulation

can also help find the design tradeoffs to determine the best modification for gears which must operate over a range of loads.

2. High-contact-ratio gears require less profile modification than standard low-contact-ratio gears. Excess modification has a more detrimental effect than under modification.

3. While excess modification increases dynamic load, a slight increase in modification or a longer zone of modification tends to shift the location of the peak load to a lower point on the tooth profile which reduces the tooth root stress.

4. The optimum profile modification for high-contact-ratio gears involves a tradeoff between minimum load (which affects contact stress) and minimum root (bending) stress.

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16. Abstract  This paper presents a computer simulation for the dynamic response of high-contact-ratio spur gear transmissions. High contact ratio gears have the potential to produce lower dynamic tooth loads and minimum root stress but they can be sensitive to tooth profile errors. The analysis presented in this paper examines various profile modifications under realistic loading conditions. The effect of these modifications on the dynamic load (force) between mating gear teeth and the dynamic root stress is presented. Since the contact stress is dependent on the dynamic load, minimizing dynamic loads will also minimize contact stresses. This paper shows that the combination of profile modification and the applied load (torque) carried by a gear system has a significant influence on gear dynamics. The ideal modification at one value of applied load will not be the best solution for a different load. High-contact-ratio gears were found to require less modification than standard low-contact-ratio gears. High-contact-ratio gears are more adversely affected by excess modification than by under modification. In addition, the optimal profile modification required to minimize the dynamic load (hence the contact stress) on a gear tooth differs from the optimal modification required to minimize the dynamic root (bending) stress. Computer simulation can help find the design tradeoffs to determine the best profile modification to satisfy the conflicting constraints of minimizing both the load and root stress in gears which must operate over a range of applied loads.			
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